48

ANALYSIS OF ANISOTROPIC CURVED SHELLS

COMPUTER PROGRAM USER'S MANUAL

v1.0

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INTRODUCTION

PROGRAM DESCRIPTION

The advent of composite structures in aerospace designs and the increasing affordability of those materials have increased the number of applications and the widespread need of analytical tools for the analysis and design of composite structures.

The software here presented is able to analyze (static and linear buckling analyses) stiffened composite panels, flat and cylindrical shells.

UNITS

The program has no built-in units; therefore any consistent unit system is appropriate. This means that, for example, to be consistent with the English system, the following have to be considered [F] = lb. (pound-force), [L] = in. (inches), [t] = sec(ond), then mass has to be in [M]=lb-sec^2/in. In a similar way a typical International System derived units often used in structural analysis is [F] = N (Newton), [L] = mm (millimeter), [t] = sec(ond), then mass has to be in [M]=N-sec^2/mm = tons.

COORDINATE SYSTEMS, SHELL MODEL DESCRIPTION, SIGN CONVENTION

In the following figures the coordinate systems for shell definition and parameters are shown. The positive load sign convention is depicted in Figure 2. Origin of coordinate system is the shell's lower left corner.

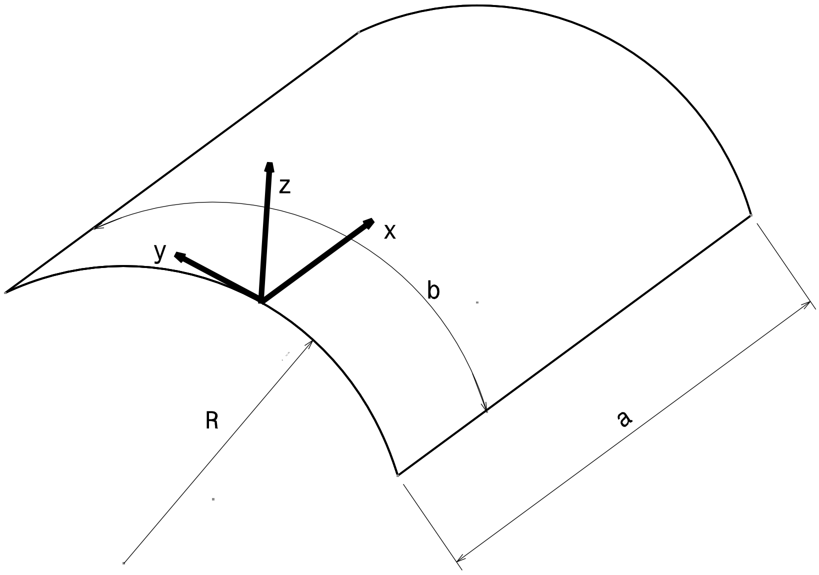


Figure 1. Shell geometry

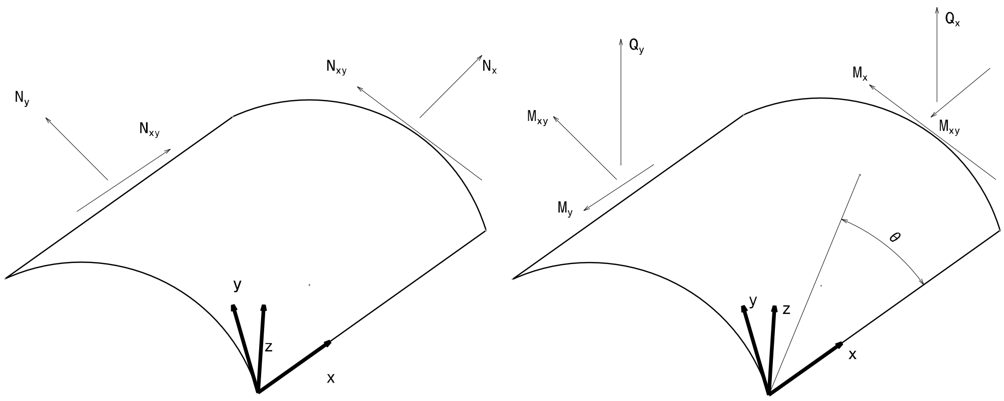


Figure 2. Sign convention for positive loads

TYPES OF ANALYSIS AVAILABLE

The program includes the following capabilities:

A. Types of Analysis

1. Static deflection and strength under complicated variations of edge and lateral loads with complicated support conditions

2. Elastic stability under complicated edge loads

~~3. Natural frequencies and mode shapes.~~ *Functionality not validated yet.*

B. Geometry

1. Flat panel

2. Cylindrically curved panel

~~3. Full cylinder (especially orthotropic only).~~ *Functionality not validated yet.*

C. Construction

1. Sheet with discrete rings and stringers

2. Sandwich with discrete rings and stringers (neglecting core shear).

D. Material - Linearly Elastic

1. Panel - layered anisotropic

2. Stiffeners - orthotropic.

E. Boundary Conditions

1. All combinations of clamped and simply supported; some combinations with free edges

2. Elastic moment restraint on opposite edges.

MODEL DESCRIPTION

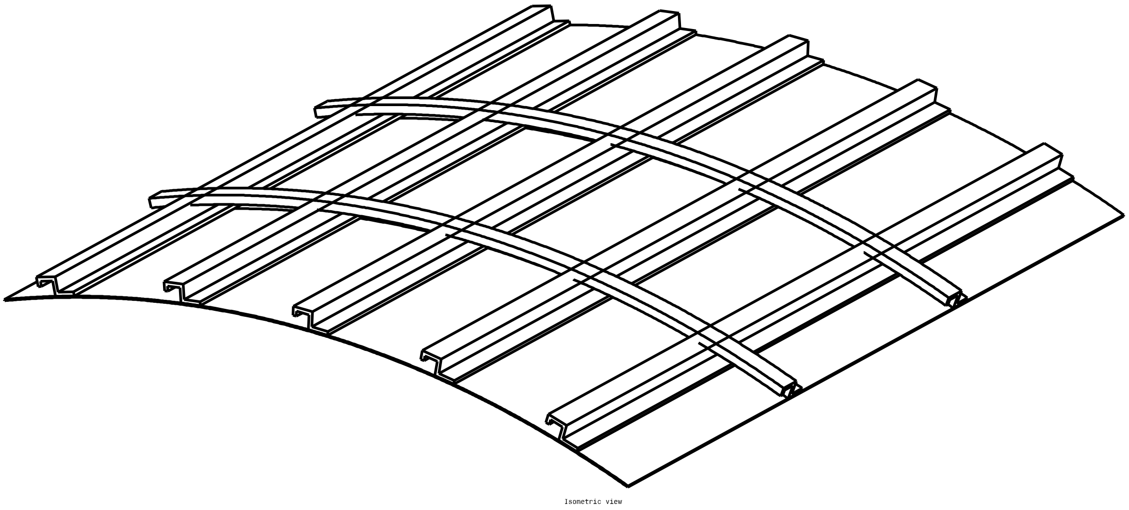


Figure 3. Geometry of discretely stiffened shell

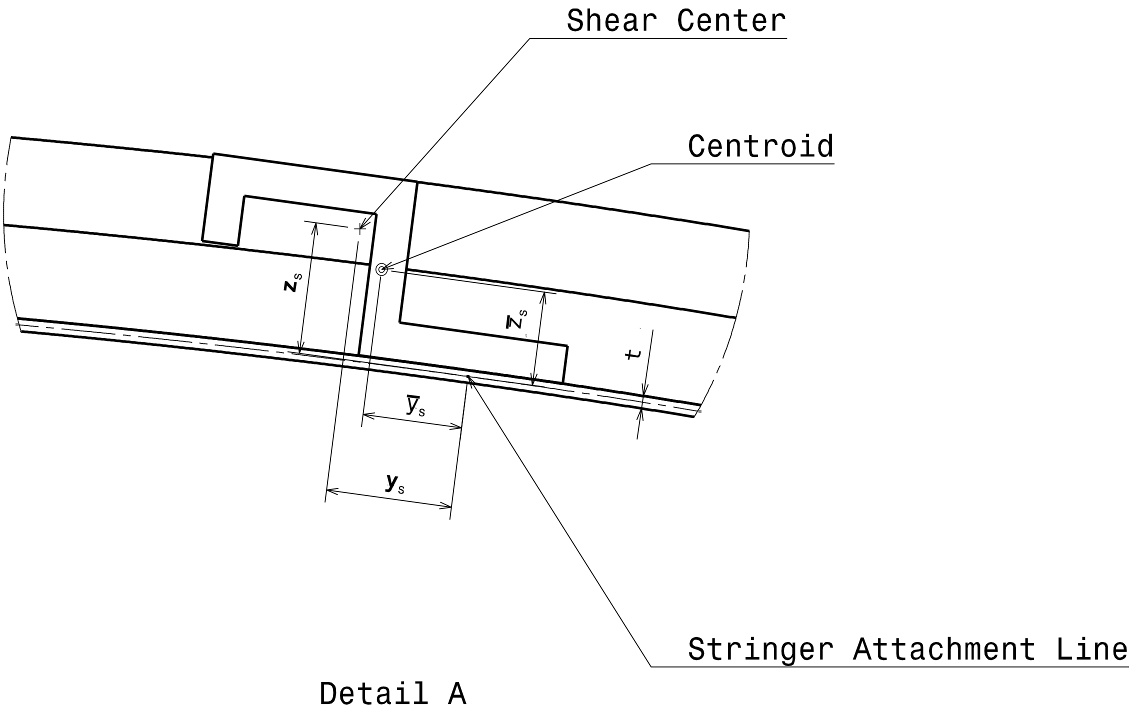


Figure 4. External stringer detail

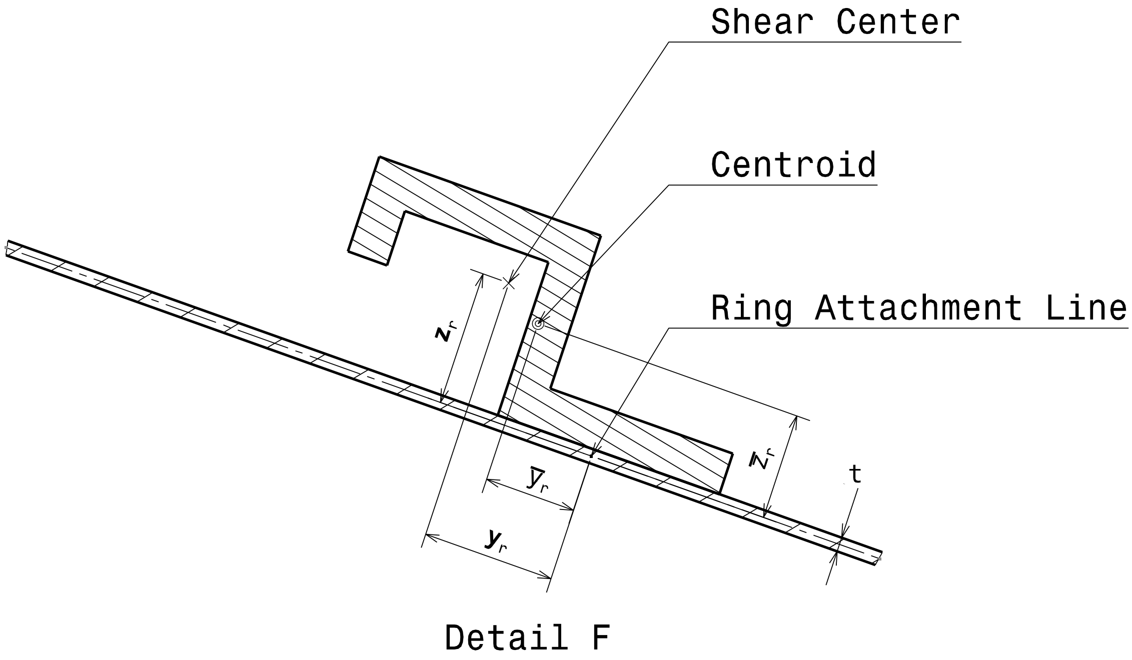


Figure 5. External ring (frame) detail

USER'S INSTRUCTIONS

A description of the data Cards follows:

Card 1. Title

Printed with the output. Any Fortran characters may be used. (1 card only.)

Card 2. IFLAGD, IFLAGB, IFLAGW, IBCX, IBCY, NTX, NTY, ITX, ITY, NMODES, IMATL, NPLYS, IREACT, IOUT, IEDGE, NPNX, NPNY, IPRTN, NQTX, NQTY, IPRTQ, NSTRNG, NRING, NLMASS, NPTLDS, NPTMOM, NLNMOM, NPTSUP, NLNSPR, INTPRT, IFLEX (31 integers).

IFLAGD = +1, if doing a dynamics problem

= +0, otherwise.

IFLAGB = +1, if 1 buckling eigenvalue is desired

= +2, if 2 buckling values are desired (as for shear buckling)

= +3, if 1 buckling eigenvalue and an imperfection sensitivity analysis are desired

= +4, if 2 buckling eigenvalues and an imperfection sensitivity analysis are desired

= +0, otherwise.

IFLAGW = +1, if doing a deflection analysis with lateral pressure, *q*

= +2, if doing a deflection analysis with no lateral pressure, q

= +0, otherwise.

IBCX is a tag for the boundary condition in the x-direction.

= +1, for clamped-simply supported

= +2, for simply supported-simply supported

= +3, for clamped-clamped

= +4, for clamped-free

= +5, for simply supported-free

= +6, for free-free

= +7, for elastic restraint. ($w_{,xx} = \alpha_x w_{,x}|_{x=0}$, $w_{,xx} = \beta_x w_{,x}|_{x=a}$))

IBCY is a tag for the boundary condition in the y-direction.

= +0, for a full cylinder

= +1, for clamped-simply supported

= +2, for simply supported-simply supported

= +3, for clamped-clamped

= +4, for clamped-free

= +5, for simply supported-free

= +6, for free-free

= +7, for elastic restraint exactly the same as that in the x-direction

= +8, other elastic restraint. ($w_{,yy} = \alpha_y w_{,y}|_{y=0}$, $w_{,yy} = \beta_y w_{,y}|_{y=b}$)

NTX = Number of terms in the assumed series for u, v, and w, in the x-direction. **1 ≤ NTY ≤ 10**.

NTY = Number of terms in the assumed series for u, v, and w, in the y-direction. **1 ≤ NTY ≤ 10**.

Note: Although the upper limit on each of the above two numbers is ten, the limit on the size of the matrices generated using them is 150. This means that NTX * NTY ≤ 50.

ITX = The beginning term in the assumed series for u, v, and w. This number sets the range of m (axial wave number) to be considered in the analysis, such that ITX ≤ M ≤ ITX + NTX -1. The range on ITX is 1≤ITX≤20.

ITY = The beginning term in the assumed series for u, v, and w. This number sets the range of n (circumferential wave number) to be considered in the analysis, such that ITY≤ N ≤ ITY + NTY -1. The range on ITY is 1≤ITY≤20.

NMODES = Number of mode shapes to be calculated in a natural frequency problem. 1≤NMODES≤20.

= +0, for a buckling or lateral loads problem.

IMATL = +1, for an isotropic material

= +2, for a laminate with constant ply properties

= +3, for a laminate with variable ply properties

= +4, for a sandwich with orthotropic facings.

NPLYS = Number of plies in the laminate 1≤NPLYS≤40. For an isotropic material, NPLYS = +1. For a sandwich, NPLYS = +3.

IREACT = +1 if the reactions (at the corners, or along the edges of the panel, or at elastic supports) are desired.

= +0, otherwise.

IOUT = An indicator that controls how much output is given and also controls whether a lamina strength analysis is performed. Each of the following output quantities is printed at 625 points over the panel, with the x = 0 axis across the top and the y = 0 axis down the left hand side.

= +1, for printing the normal deflection, w, only

= +2, for printing w, u, and v (mid-surface deflections)

= +3, for printing w, u, v, ε_x^0, ε_y^0, γ_{xy}^0 (mid-surface strains) and κ_x, κ_y, κ_{xy} (curvatures)

= +4, for printing w, u, v, M_x, M_y, M_{xy} (moment resultants), Q_x , Q_y , (transverse shear resultants), and σ_x, σ_y, τ_{xy}, σ_{VM} (stresses, only for isotropic or sandwich, Von Mises stress only for isotropic material)

= +5, for printing w, u, v, M_x, M_y, M_{xy}, Q_x, Q_y, σ_x, σ_y, τ_{xy}, σ_{VM}, ε_x^0, ε_y^0, γ_{xy}^0, κ_x, κ_y, κ_{xy}

= +6, for printing w, σ_x, σ_y, τ_{xy}

= +7, for printing w, ε_1, ε_2, γ_{12}, (strains in lamina axes for each ply), RF_1, RF_2, RF_{12} (reserve factors for each ply according to the maximum strain theory)

= +8, for printing w, σ_x, σ_y, τ_{xy}, σ_{VM}, ε_1, ε_2, γ_{12}, RF_1, RF_2, RF_{12}

= +9, for printing w, u, v, M_x, M_y, M_{xy}, Q_x, Q_y, ε_x^0, ε_y^0, ε_{xy}^0, κ_x, κ_y, κ_{xy}, σ_x, σ_y, τ_{xy}, σ_{VM}, ε_1, ε_2, γ_{12}, RF_1, RF_2, RF_{12}

IEDGE = +1, if edge loads are to be input

= +2, if cylinder end-loads (force, torque, bending moment) are to be input.

= +0, otherwise.

NPNX = Number of terms in the edge loads expressions in the x-direction. 1≤NPNX≤10.

= +0, if IEDGE = +0 or +2.

NPNY = Number of terms in the edge loads expressions in the y-direction. 1≤NPNX≤10.

= +0, if IEDGE = +0 or +2.

IPRTN = +1, if the distributions of the edge loads are to be printed at quarter points of the panel.

= +0, otherwise.

NQTX = Number of terms in the distributed lateral loads expression in the x-direction. 1≤NQTX≤10.

= +0, if IFLAGW = +0 or +2.

NQTY = Number of terms in the distributed lateral loads expression in the y-direction. 1≤NQTY≤10.

= +0, if IFLAGW = +0 or +2.

IPRTQ = +1, if the distribution of the lateral loads is to be printed at quarter points of the panel.

= +0, otherwise.

NSTRNG = Number of stringers. 0≤NSTRNG≤100. (For equally-spaced identical stringers, precede number by a minus sign.)

NRING = Number of rings. 0≤NRING≤50. (For equally-spaced identical rings, precede number by a minus sign.)

NLMASS = Number of lumped masses. 0≤NLMASS≤50.

NPTLDS =Number of concentrated normal loads. 0≤NPTLDS≤50.

NPTMOM = Number of concentrated point moments. 0≤NPTMOM≤50.

NLNMOM = Number of concentrated line moments. 0≤NLNMOM≤50.

NPTSUP = Number of point spring supports. 0≤NPTSUP≤50.

NLNSPR = Number of line spring supports. 0≤NLNSPR≤50.

INTPRT = +1, if the values of the calculated integrals, the matrices generated is to be printed.

= +0, otherwise. (Usually, INTPRT = +0).

IFLEX = Number of points for which influence coefficients are desired.

Card 3. AA, [BB], RR, [MU]

AA = Dimension in the x-direction

BB = Dimension in the y-direction (Note: This is not input for a full cylinder.)

RR = Radius of panel.

MU = Standard deviation of panel thickness.

Card 4. [ALFAX, BETAX], [ALFAY, BETAY]

ALFAX = The constant describing the elastic restraint on the edge x = 0. $w_{,xx}$ = (ALFAX) $w_{,x}$.

BETAX = The constant describing the elastic restraint on the edge x = a. $w_{,xx}$ = (-BETAX) $w_{,x}$.

ALFAY = The constant describing the elastic restraint on the edge y = 0. $w_{,yy}$ = (ALFAY) $w_{,y}$.

BETAY = The constant describing the elastic restraint on the edge y = b. $w_{,yy}$ = (-BETAY) $w_{,y}$.

The elastic restraint constants are only input as needed, and if the y-direction quantities are identical to those in the x-direction, only ALFAX and BETAX need be input. All of these constants are input as positive for positive restraint.

Card 5. *E, ν, T*

E = Young's modulus, [F]*[L]**(-2)

ν = Poisson's ratio, dimensionless

T = Panel thickness, [L].

Card 6. *EC (1), EC(2), EC(3), ET(1), ET(2), ET(3)*

EC(1) = Compressive strain allowable in the 1-direction, [L]/[L].

EC(2) = Compressive strain allowable in the 2-direction, [L]/[L].

EC(3) = Negative shear strain allowable, [L]/[L].

ET(1) = Tensile strain allowable in the 1-direction, [L]/[L].

ET(2) = Tensile strain allowable in the 2-direction, [L]/[L].

ET(3) = Positive shear strain allowable, [L]/[L].

Card 7. E1, E2, G, ν_{12}, H, (θ_i, i = 1, 2, …, NPLYS)

E1 = Modulus in the 0° direction, [F]*[L]**(-2).

E2 = Modulus in the 90° direction, [F]*[L]**(-2).

G = In-plane shear modulus, [F]*[L]**(-2).

ν_{12} = Major Poisson's ratio, dimensionless.

H = Thickness of each ply, [L]

θ_i = Orientation of the i^{th} ply, starting with the bottom or inner ply, degrees.

Card 8. (E1), (E2), G, ν_{12}, θ_i, [$EC(1)_i$, $EC(2)_i$, $EC(3)_i$, $ET(1)_i$,$ET(2)_i$, $ET(3)_i$], *i* = 1,…, NPLYS

$E1_i$ = Modulus in the 0° direction of the i^{th} ply, [F]*[L]**(-2)

$E2_i$ = Modulus in the 90° direction of the i^{th} ply, [F]*[L]**(-2)

G_i = Shear modulus of the i^{th} ply, [F]*[L]**(-2)

$(\nu_{12})_i$ = Major Poisson's ratio of the i^{th} ply, dimensionless

H_i = Thickness of the i^{th} ply, [L].

θ_i = Orientation of the i^{th} ply, degrees.

(The following allowables are input only if a strength analysis is being performed, IOUT≥7.)

$EC(1)_i$ = Compressive strain allowable in the 1-direction for the i^{th} ply, [L]/[L].

$EC(2)_i$ = Compressive strain allowable in the 2-direction for the i^{th} ply, [L]/[L].

$EC(3)_i$ = Negative shear strain allowable in the 1-2 plane for the i^{th} ply, [L]/[L].

$ET(1)_i$ = Tensile strain allowable in the 1-direction for the i^{th} ply, [L]/[L].

$ET(2)_i$ = Tensile strain allowable in the 2-direction for the i^{th} ply, [L]/[L].

$ET(3)_i$ = Positive shear strain allowable in the 1-2 plane for the i^{th} ply, [L]/[L].

Card 9. E1, E2, G, ν_{12}, H

E1 = Inner (outer) facing modulus in the 0°direction, [F]*[L]**(-2).

E2 = Inner (outer) facing modulus in the 90° direction, [F]*[L]**(-2).

G = Inner (outer) facing shear modulus, [F]*[L]**(-2).

ν_{12} = Inner (outer) facing major Poisson's ratio, dimensionless.

H = Inner (outer) facing thickness, [L].

(If a strength analysis is not being performed, Card 9 is now repeated for the outer facing properties. If a strength analysis is being performed, Cards 10 and 11 for the inner facing are now input, and then Cards 9, 10 and 11 are input for the outer facing.)

Card 10. EC(1), EC(2), EC(3), ET(1), ET(2), ET(3), MCHK

EC(1) = Inner (outer) facing compressive strain allowable in the 1-direction, [L]/[L].

EC(2) = Inner (outer) facing compressive strain allowable in the 2-direction, [L]/[L].

EC(3) = Inner (outer) facing negative shear strain allowable in the 1-2 plane, [L]/[L].

ET(1) = Inner (outer) facing tensile strain allowable in the 1-direction, [L]/[L].

ET(2) = Inner (outer) facing tensile strain allowable in the 2-direction, [L]/[L].

EC(3) = Inner (outer) facing positive shear strain allowable in the 1-2 plane, [L]/[L].

MCHK = Number of orientations to be checked in the strength analysis of the inner (outer) facing. 1≤MCHK≤10.

Card 9. E1, E2, G, ν_{12}, H

E1 = Inner (outer) facing modulus in the 0° direction, [F]*[L]**(-2).

E2 = Inner (outer) facing modulus in the 90 direction, [F]*[L]**(-2).

G = Inner (outer) facing shear modulus, [F]*[L]**(-2).

ν_{12} = Inner (outer) facing major Poisson's ratio, dimensionless.

H = Inner (outer) facing thickness, [L].

(If a strength analysis is not being performed, Card 9 is now repeated for the outer facing properties. If a strength analysis is being performed, Cards 10 and 11 for the inner facing are now input, and then Cards 9, 10 and 11 are input for the outer facing.)

Card 10. EC(1), EC(2), EC(3), ET(1), ET(2), ET(3), MCHK

EC(1) = Inner (outer) facing compressive strain allowable in the 1-direction, [L]/[L].

EC(2) = Inner (outer) facing compressive strain allowable in the 2-direction, [L]/[L].

EC(3) = Inner (outer) facing negative shear strain allowable in the 1-2 plane, [L]/[L].

ET(1) = Inner (outer) facing tensile strain allowable in the 1-direction, [L]/[L].

ET(2) = Inner (outer) facing tensile strain allowable in the 2-direction, [L]/[L].

ET(3) = Inner (outer) facing positive shear strain allowable in the 1-2 plane, [L]/[L].

MCHK = Number of orientations to be checked in the strength analysis of the inner (outer) facing. $1 \leq MCHK \leq 10$.

Card 11. $ANGCHK_i$, i = 1, MCHK

$ANGCHK_i$= Orientations to be checked in the strength analysis of the inner (outer) facing, degrees.

Card 12. H_c, (s, Ec, G_L, G_W)

H_c = Core thickness, [L]

s, E_c, G_L, G_W **only if IOUT≥7**

s = Honeycomb core cell size, [L]

E_c = Core modulus, [F]/[L]**2.

G_L = Core shear modulus in ribbon direction (L direction), [F]/[L]**2.

G_W = Core shear modulus in transverse (expansion) direction (W direction), [F]/[L]**2.

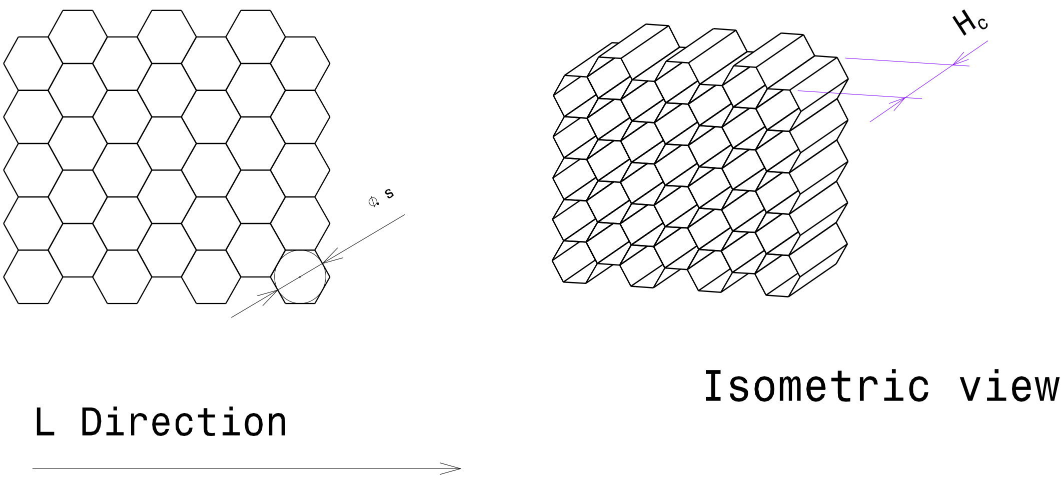


Figure 6. Honeycomb core geometrical definitions

Card 13. [YSTRNG], YBAR, ZBAR, AS, XIYYS, XIYZS, XIZZS, ES, GJS, RHOS

See Figure XX for symbol explanation

YSTRNG = Distance of longitudinal stiffener from y = 0. For variable stiffener spacing only.

YBAR = Location of stringer centroid in the y-direction with respect to its line of attachment to the shell, [L]

ZBAR = Location of stringer centroid in the z-direction with respect to the middle surface of the shell at the line of attachment, [L]

AS = Stringer cross-sectional area, [L]**2.

XIYYX = Moment of inertia of the stringer area about the mid-surface y- axis at the line of attachment, [L]**4.

XIYZS = Product of inertia of the stringer area about the mid-surface y-z axis at the line of attachment, [L]**4.

XIZZS = Moment of inertia of the stringer area about the z-axis at the line of attachment, [L]**4.

ES = Stringer modulus of elasticity, [F]*[L]**(-2).

G*JS = Stringer torsional stiffness, [F]*[L]**(+2).

RHOS = Average density of stringer material, [F]*[t]**(2)/[L]**(4).

Card 13 is repeated 'NSTRNG' times, unless equally-spaced identical stringers were specified.

Card 14. [XRING], XBARR, ZBARR, AR, XIXXR, XIZZR, ER, GJR, RHOR

See Figure XX for symbol explanation

XRING = Distance of circumferential stiffener from x = 0. For unequally spaced rings.

XBARR = Location of ring centroid in the x-direction with respect to its line of attachment to the shell, [L].

ZBARR = Location of ring centroid in the z-direction with respect to the middle surface of the shell at the line of attachment, [L].

AR = Ring cross-sectional area, [L]**2.

XIXXR = Moment of inertia of the ring area about the mid-surface x-axis at the line of attachment, [L]**4.

XIXZR = Product of inertia of the ring area about the mid-surface x-z axis at the line of attachment, [L]**4.

XIZZR = Moment of inertia of the ring area about the z-axis at the line of attachment, [L]**4

ER = Ring modulus of elasticity, [F]*[L]**(-2).

G*JR = Ring torsional stiffness, [F]*[L]**(+2).

RHOR = Average density of ring material, [F]*[t]**(2)/[L]**(4).

Card 14 is repeated ‘NRING’ times unless equally-spaced identical rings were specified.

Card 15. DENSE

DENSE = Average material density of the shell material, such that (DENSE)*(Vol. of shell) = Mass of shell), [F]*[t]**(2)/[L]**(4).

Card 16. IX, IY, PMASS

IX = Grid coordinate in x-direction at which lumped mass is located, 1≤IX≤25.

IY = Grid coordinate in y-direction at which lumped mass is located, 1≤IY≤25.

PMASS = Mass, [F]*[t]**(2)/[L].

Card 16 is repeated 'NLMASS' times.

Card 17. PX(1,1), PY(1,1), PXY(1,1), PX(2,1), PY(2,1), PXY(2,1), PX(I, J), PY(I,J), PXY(I,J), I= 1,2...NPNX, J = 1,2…,NPNY

The applied in-plane stress resultants are described by the relations:

$$N_x(x,y) = \sum_{I=1}^{NPNX} \sum_{J=1}^{NPNY} P_x(I,J) \left(\frac{x}{a}\right)^{I-1} \left(\frac{y}{b}\right)^{J-1}$$

$$N_y(x,y) = \sum_{I=1}^{NPNX} \sum_{J=1}^{NPNY} P_y(I,J) \left(\frac{x}{a}\right)^{I-1} \left(\frac{y}{b}\right)^{J-1}$$

$$N_{xy}(x,y) = \sum_{I=1}^{NPNX} \sum_{J=1}^{NPNY} P_{xy}(I,J) \left(\frac{x}{a}\right)^{I-1} \left(\frac{y}{b}\right)^{J-1}$$

Note: Tension stress resultants are taken as positive.

TORQUE = Torque applied to cylinder, [F]*[L].

BNDMOM = Bending moment applied to cylinder, [F]*[L].

Card 19. Q(1,1), Q(2,1), Q(3,1), ...Q(I,J), 1=1, ..., NQTX, J = 1,2..., NQTY

The distributed lateral load is described by the relation

$$q(x,y) = \sum_{I=1}^{NQTX} \sum_{J=1}^{NQTY} Q(I,J) \left(\frac{x}{a}\right)^{I-1} \left(\frac{y}{b}\right)^{J-1}$$

Note: positive loads are in the positive z-direction.

Card 20. IX, IY, PC

IX = Grid coordinate in x-direction, 1≤IX≤25.

IY = Grid coordinate in y-direction, 1≤IY≤25.

PC = Concentrated load, [F].

Card 20 is repeated 'NPTLDS' times.

Card 21. IX, IY, ITAG, FC

IX = Grid coordinate in x-direction, 1≤IX≤25.

IY = Grid coordinate in the y-direction, 1≤IY≤25.

ITAG = +1, if the moment is about the x-axis in a vector sense (right-hand rule)

= +2, if the moment is about the y-axis.

FC = Moment, [F]*[L].

Card 21 is repeated 'NPTMOM' times.

Card 22. ITAG, IDIST, PLMOM

ITAG = +1, if the line moment is parallel to the x-axis.

= +2, if the line moment is parallel to the y-axis.

IDIST = Number of grid lines away from the x = 0 or y = 0 axis. 1≤ IDIST≤25.

PLMOM = Line moment per unit of length, [F].

Card 22 is repeated 'NLNMOM' times.

Card 23. IX, IY, PKC

IX = Grid coordinate in x-direction. $1 \le IX \le 25$.

IY = Grid coordinate in y-direction. $1 \le IY \le 25$.

PKC = Spring constant, [F]/[L].

Card 23 is repeated 'NPTSUP' times.

Card 24. ITAG, IDIST, PLINE

ITAG = +1, if the line spring is parallel to the x-axis.

= +2, if the line spring is parallel to the y-axis.

IDIST = Number of grid lines away from the x=0 or y=0 axis. $1 \le MIDIST \le 25$.

PLINE = Spring constant per unit length, [F]/[L].

Card 24 is repeated 'NLNSPR' times.

Card 25. XP (I), YP (I), I=1, IFLEX

XP (I) = X-coordinate (in %) of I^{th} flexibility matrix control point.

YP (I) = Y-coordinate (in %) of I^{th} flexibility matrix control point.

The following list is an example of an input file for the analysis of a composite curved shell reinforced with T-section stringers equally spaced.

`posicoss p14 - UNITS = N, mm`	*Title*
`0,2,0,3,3,7,7,1,1,0,2,8,0,2,1,1,1,1,0,0,0,4,0,0,0,0,0,0,0,0,0`	IFLAGD, IFLAGB, IFLAGW, IBCX, IBCY, NTX, NTY, ITX, ITY, NMODES, IMATL, NPLYS, IREACT, IOUT, IEDGE, NPNX, NPNY, IPRTN, NQTX, NQTY, IPRTQ, NSTRNG, NRING, NLMASS, NPTLDS, NPTMOM, NLNMOM, NPTSUP, NLNSPR, INTPRT, IFLEX *analysis parameters*
`660.,419.,947.,0.`	A, B, R, MU *shell geometry*
`146535.,9720.,6054.,.34,.125,90,+45,-45,0,0,-45,+45,90`	E1, E2, G12, v12, h, θi *lamina mechanical properties and lay-up*

`52.375,0.,-4.216,88.5,3129.875,0.,4941.125,81178.73,2525710,1.6e-9`
`157.125,0.,-4.216,88.5,3129.875,0.,4941.125,81178.73,2525710,1.6e-9`
`261.875,0.,-4.216,88.5,3129.875,0.,4941.125,81178.73,2525710,1.6e-9`
`366.625,0.,-4.216,88.5,3129.875,0.,4941.125,81178.73,2525710,1.6e-9`
	YSTRNG, YBAR, ZBAR, AS, XIYYS, XIYZS, XIZZS, ES, GJS, RHOS *for each of the four stringers*

`-178.,0.,0.`	Px(1,1), Py(1,1), Pxy(1,1) *loads applied to the shell's edge (= Nx, Ny, Nxy as they are constant in this example)*

A more detailed explanation of the analysis parameters line follows:

`0,2,0,`	Buckling analysis, 2 eigenvalues desired
`3,3,`	Boundary conditions: Clamped in x-direction (both sides) and Clamped in y-direction (both sides)
`7,7,`	7 terms in the assumed series in x-direction 7 terms in the assumed series in y-direction
`1,1,`	Beginning term in the assumed series x-direction = 1 Beginning term in the assumed series y-direction = 1
`0,`	Buckling problem, hence modes for natural frequencies = 0
`2,`	Laminate with constant ply properties
`8,`	Number of plies in the laminate
`0,`	Reactions are not desired in this example
`2,`	*w, u* and *v*, mid-surface deflections is the only output desired in this example
`1,1,1,1,`	Edge loads are input One term in x-direction One term in y-direction Edge loads will be printed in output
`0,0,0,`	No term in x-direction for lateral loads No term in y-direction for lateral loads Lateral loads will not be printed in output (there are no lateral loads in this example)
`4,0,`	Four stringers not equally spaced Zero rings
`0, 0, 0, 0, 0, 0,`	No lumped masses No concentrated normal loads No concentrated point moments No concentrated line moments No point spring supports No line spring supports
`0,`	No intermediate output is desired
`0`	No influence coefficients desired

OUTPUT DATA DESCRIPTION

First block of output is a punch of input file completely labeled for easy reading.

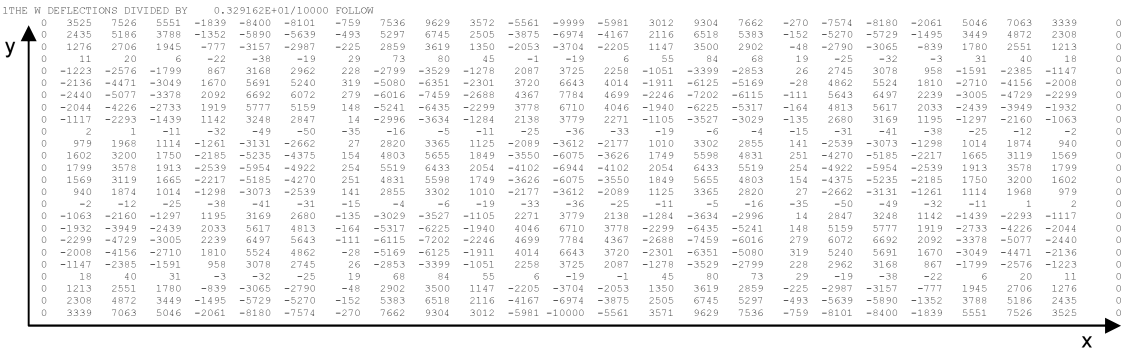
```
posicoss p14 - UNITS = N, mm
0THE BOUNDARY CONDITIONS AT X=0 AND X=A ARE
 CLAMPED, CLAMPED
0THE BOUNDARY CONDITIONS AT Y=0 AND Y=B ARE
 FREE, FREE
0THERE ARE  7 MODES IN THE X DIRECTION, STARTING WITH M =  1 .
 THERE ARE  7 MODES IN THE Y DIRECTION, STARTING WITH N =  2 .
0THE STIFFNESS MATRIX SIZE IS 147 BY 147
0A STABILITY SOLUTION WILL BE SOUGHT
0A =           660.00000
0B =           419.00000
0R =           947.00000
0MU =            0.00000
0FOR THE  8 PLY LAMINATE
0E1 =        0.146535E+06
0E2 =        0.972000E+04
0G =        0.605400E+04
0NU12 = 0.3400
0H(I) =   0.1250
0T =    1.0000
0THE ORIENTATIONS ARE
    90.0000

    45.0000

   -45.0000

     0.0000

     0.0000

   -45.0000

    45.0000

    90.0000

1THE CONSTITUTIVE MATRIX IS


        62907.99        19153.56            0.00            0.00            0.00            0.00
        19153.56        62907.99            0.00            0.00            0.00            0.00
            0.00            0.00        21877.22            0.00            0.00            0.00
            0.00            0.00            0.00         2258.19         1348.89          538.56
            0.00            0.00            0.00         1348.89         8720.95          538.56
            0.00            0.00            0.00          538.56          538.56         1575.86
0THE LAMINATE PROPERTIES ARE
0EX =    0.570763E+05   EY =    0.570763E+05   G =    0.218772E+05   NUXY =  0.3045   NUYX =  0.3045
0THE STRINGER PROPERTIES FOLLOW --
0L      Y      YBAR   ZBAR   AREA      IYY         IYZ         IZZ          E          GJ          RHO

   1    52.38   0.00  -4.22  88.50  3.1299E+03  0.0000E+00  4.9411E+03  8.1179E+04  2.5257E+06  1.6000E-09
   2   157.13   0.00  -4.22  88.50  3.1299E+03  0.0000E+00  4.9411E+03  8.1179E+04  2.5257E+06  1.6000E-09
   3   261.88   0.00  -4.22  88.50  3.1299E+03  0.0000E+00  4.9411E+03  8.1179E+04  2.5257E+06  1.6000E-09
   4   366.63   0.00  -4.22  88.50  3.1299E+03  0.0000E+00  4.9411E+03  8.1179E+04  2.5257E+06  1.6000E-09
0PX(I,J) FOLLOWS
 -1.7800E+02
0PY(I,J) FOLLOWS
  0.0000E+00
0PXY(I,J) FOLLOWS
  0.0000E+00
```

Most of the output is labeled. The output is of two types: data like the eigenvalue in buckling problems

```
0THE BUCKLING EIGENVALUE IS       0.4782526 FOR M =  7, N =  4.
```

and 2D maps for variables like displacements, strains, stresses, etc. The coordinate system applicable to these maps is shown in Figure XX, relative to the shell coordinate system. (0, 0) is the ***lower left*** corner and (a, b) is the ***upper right*** corner. There is a map of 25 by 25 points equally distributed in x and y-directions. The maps are normalized to the maximum value in the map.

These maps can easily be used to create 3D color plots. The following is an example of the vertical deflection output (*w*)



```
1THE W DEFLECTIONS DIVIDED BY    0.329162E+01/10000 FOLLOW
     0  3525  7526  5551 -1839 -8400 -8101  -759  7536  9629  3572 -5561 -9999 -5981  3012  9304  7662  -270 -7574 -8180 -2061  5046  7063  3339       0
     0  2435  5186  3788 -1352 -5890 -5639  -493  5297  6745  2505 -3875 -6974 -4167  2116  6518  5383  -152 -5270 -5729 -1495  3449  4872  2308       0
     0  1276  2706  1945  -777 -3157 -2987  -225  2859  3619  1350 -2053 -3704 -2205  1147  3500  2902   -48 -2790 -3065  -839  1780  2551  1213       0
     0    11    20     6   -22   -38   -19    29    73    80    45    -1   -19     6    55    84    68    19   -25   -32    -3    31    40    18       0
     0 -1223 -2576 -1799   867  3168  2962   228 -2799 -3529 -1278  2087  3725  2258 -1051 -3399 -2853    26  2745  3078   958 -1591 -2385 -1147       0
     0 -2136 -4471 -3049  1670  5691  5240   319 -5080 -6351 -2301  3720  6643  4014 -1911 -6125 -5169   -28  4862  5524  1810 -2710 -4156 -2008       0
     0 -2440 -5077 -3378  2092  6692  6072   279 -6016 -7459 -2688  4367  7784  4699 -2246 -7202 -6115  -111  5643  6497  2239 -3005 -4729 -2299       0
     0 -2044 -4226 -2733  1919  5777  5159   148 -5241 -6435 -2299  3778  6710  4046 -1940 -6225 -5317  -164  4813  5617  2033 -2439 -3949 -1932       0
     0 -1117 -2293 -1439  1142  3248  2847    14 -2996 -3634 -1284  2138  3779  2271 -1105 -3527 -3029  -135  2680  3169  1195 -1297 -2160 -1063       0
     0     2     1   -11   -32   -49   -50   -35   -16    -5   -11   -25   -36   -33   -19    -6    -4   -15   -31   -41   -38   -25   -12    -2       0
     0   979  1968  1114 -1261 -3131 -2662    27  2820  3365  1125 -2089 -3612 -2177  1010  3302  2855   141 -2539 -3073 -1298  1014  1874   940       0
     0  1602  3200  1750 -2185 -5235 -4375   154  4803  5655  1849 -3550 -6075 -3626  1749  5598  4831   251 -4270 -5185 -2217  1665  3119  1569       0
     0  1799  3578  1913 -2539 -5954 -4922   254  5519  6433  2054 -4102 -6944 -4102  2054  6433  5519   254 -4922 -5954 -2539  1913  3578  1799       0
     0  1569  3119  1665 -2217 -5185 -4270   251  4831  5598  1749 -3626 -6075 -3550  1849  5655  4803   154 -4375 -5235 -2185  1750  3200  1602       0
     0   940  1874  1014 -1298 -3073 -2539   141  2855  3302  1010 -2177 -3612 -2089  1125  3365  2820    27 -2662 -3131 -1261  1114  1968   979       0
     0    -2   -12   -25   -38   -41   -31   -15    -4    -6   -19   -33   -36   -25   -11    -5   -16   -35   -50   -49   -32   -11     1     2       0
     0 -1063 -2160 -1297  1195  3169  2680  -135 -3029 -3527 -1105  2271  3779  2138 -1284 -3634 -2996    14  2847  3248  1142 -1439 -2293 -1117       0
     0 -1932 -3949 -2439  2033  5617  4813  -164 -5317 -6225 -1940  4046  6710  3778 -2299 -6435 -5241   148  5159  5777  1919 -2733 -4226 -2044       0
     0 -2299 -4729 -3005  2239  6497  5643  -111 -6115 -7202 -2246  4699  7784  4367 -2688 -7459 -6016   279  6072  6692  2092 -3378 -5077 -2440       0
     0 -2008 -4156 -2710  1810  5524  4862   -28 -5169 -6125 -1911  4014  6643  3720 -2301 -6351 -5080   319  5240  5691  1670 -3049 -4471 -2136       0
     0 -1147 -2385 -1591   958  3078  2745    26 -2853 -3399 -1051  2258  3725  2087 -1278 -3529 -2799   228  2962  3168   867 -1799 -2576 -1223       0
     0    18    40    31    -3   -32   -25    19    68    84    55     6   -19    -1    45    80    73    29   -19   -38   -22     6    20    11       0
     0  1213  2551  1780  -839 -3065 -2790   -48  2902  3500  1147 -2205 -3704 -2053  1350  3619  2859  -225 -2987 -3157  -777  1945  2706  1276       0
     0  2308  4872  3449 -1495 -5729 -5270  -152  5383  6518  2116 -4167 -6974 -3875  2505  6745  5297  -493 -5639 -5890 -1352  3788  5186  2435       0
     0  3339  7063  5046 -2061 -8180 -7574  -270  7662  9304  3012 -5981 -10000 -5561  3571  9629  7536  -759 -8101 -8400 -1839  5551  7526  3525      0
```

PROGRAM LIMITATIONS

The ranges of the input parameters are described under INPUT DATA. The main restriction is to keep in mind the assumptions of the analysis, particularly the small-deflection assumption. If the deflections found in a lateral loads problem are greater than the panel thickness, the results are questionable. If a solution mode shape contains large contributions from the highest modal shape input, the solution is questionable, and the analysis should be rerun using the highest mode shape input as the initial term in the new analysis. Since the high-order modes are not sensitive to boundary conditions, the restriction to simply-supported or full cylinder boundary conditions will not make much difference in the results.

EXAMPLE PROBLEMS

EXAMPLE 1. Flat panel, metallic, deflection under lateral loads (uniform pressure)[1]. Assume an aluminum plate (E=10 x 10^6 psi and ν=0.3), a = 7.5”, and b= 5” with thickness t=0.1” under a uniform load p=10 psi or concentrated load P_z = 375 lbs. at plate center. Here results for simply supported on all edges will be compared.

A) Uniform load 10 psi, check w and σ_y at plate center.

(b) 1) deflection (w) Niu = 0.053”; deflection (w) 48 = 0.0527”

(a) 1) stress (σ_y) Niu = f = 12125 psi; stress (σ_y) 48 = f = 12145.6 psi

B) Concentrated load P_z = 375 lbs, check w and σ_y at plate center.

(c) 1) stress (σ_y) Niu = f = 74260 psi; stress (σ_y) 48 = f = 64108.9 psi

(d) 1) deflection (w) Niu = 0.156”; deflection (w) 48 = 0.1552”

Input and output files are available for download at the web site.

www.pmbaerospace.com/examples.html

---

[1] This example is based on Niu, M. C-Y. (1999). *Airframe Stress Analysis and Sizing*. Conmilit Press Ltd. Hong Kong, p. 191.

EXAMPLE 2. Buckling of isotropic flat plate simply supported on 4 edges, without reinforcing stiffeners.

Panel size a x b = 200 x 100 mm, thickness t= 2 mm. E = 73770 MPa, ν=0.3. Elastic limit Fty = 330.9 MPa

The buckling eigenvalue is requested as well as stresses and strains for the corresponding static load case (to evaluate Von Mises stress to check if loads are under elastic regime). An excerpt of output is shown below:

```
0THE BUCKLING EIGENVALUE IS      5.4443810 FOR M =  1, N =  1.
```

The static loads run produce the following output:

```
1THE VON MISES STRESSES ON THE LOWER SURFACE FOLLOW.    MAX. VALUE =    15.9

1THE VON MISES STRESSES ON THE UPPER SURFACE FOLLOW.    MAX. VALUE =    15.9
```

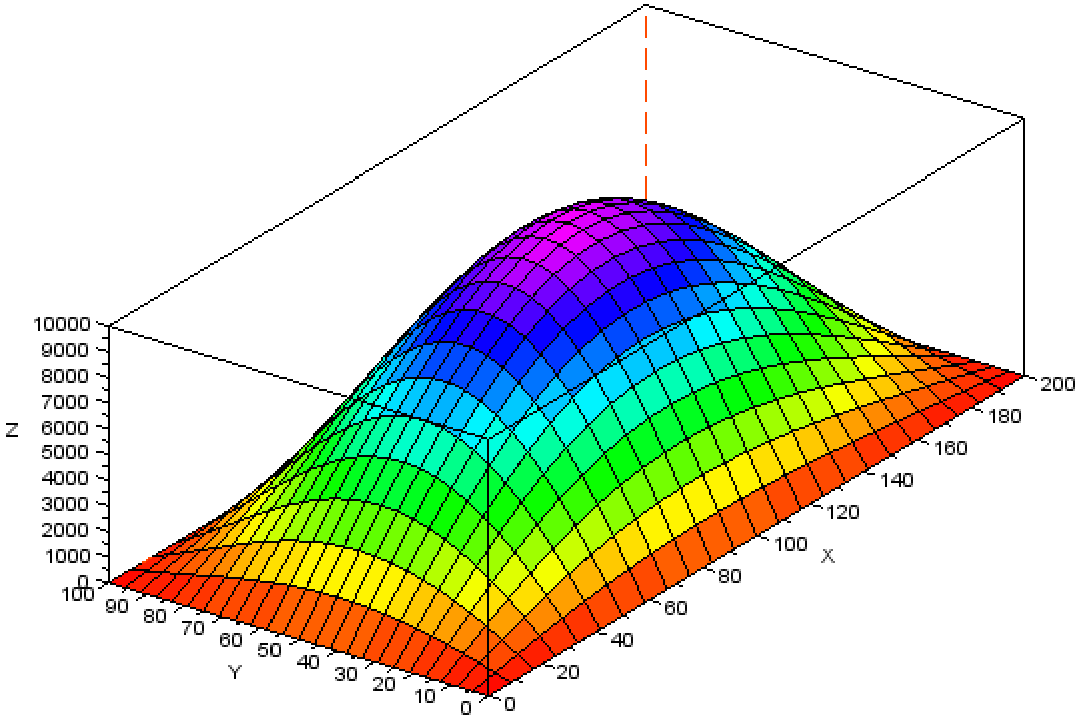


Figure 7. Mode shape (displacement w) for first buckling eigenvalue 5.44 (M=1, N=1)[2]

EXAMPLE 3. Sandwich panel with orthotropic facings with the following configuration 45/0/45/core/45/0 from outside to inside. Panel is under combined in-plane loads (Nx, Ny, Nxy) and facing failure (based on maximum strain criterion) and local stability failure modes are checked (wrinkling, dimpling and shear crimping). Boundary condition is simply supported on four edges.

---

[2] Generated using SciLab (www.scilab.org)

Wrinkling, dimpling and shear crimping allowables are computed based on faces and core characteristics provided in the input file. See output file in the following link.

```
0FOR THE INNER FACING OF THE SANDWICH

    THE DIMPLING     ALLOWABLES IN THE 1, 2, AND 12 DIRECTIONS ARE      961.71      961.71      607.17 [F]/[L]**2.

    THE WRINKLING    ALLOWABLES IN THE 1, 2, AND 12 DIRECTIONS ARE      100.50      100.50      106.08 [F]/[L]**2.

    THE SHEAR CRIMPING ALLOWABLES IN THE 1, 2, AND 12 DIRECTIONS ARE      783.43      480.17      613.33 [F]/[L]**2.
```

The reserve factors summary is presented at the end of the output file:

```
1THE MINIMUM RESERVE FACTOR FOR DIMPLING OCCURS IN LAYER 1.

 IT IS LOCATED AT X = 13, Y = 22, AND HAS A VALUE OF   40.08

1THE MINIMUM RESERVE FACTOR FOR WRINKLING OCCURS IN LAYER 3.

 IT IS LOCATED AT X =  4, Y = 22, AND HAS A VALUE OF   4.894

1THE MINIMUM RESERVE FACTOR FOR SHEAR CRIMPING OCCURS IN LAYER 3.

 IT IS LOCATED AT X =  4, Y = 22, AND HAS A VALUE OF   23.03


1THE MINIMUM RESERVE FACTOR FOR FIBER FAILURE OCCURS FOR A STRAIN IN THE 1- DIRECTION AT AN ANGLE THETA OF  45.00 DEGREES IN
LAYER  3.

 IT IS LOCATED AT X = 21, Y = 22, AND HAS A VALUE OF    23.61

1THE MINIMUM RESERVE FACTOR FOR TRANSVERSE MATRIX CRACKING OCCURS FOR A STRAIN IN THE 2- DIRECTION AT AN ANGLE THETA OF
0.00 DEGREES IN LAYER 3.

 IT IS LOCATED AT X =  4, Y = 22, AND HAS A VALUE OF    15.53

1THE MINIMUM RESERVE FACTOR FOR SHEAR MATRIX CRACKING OCCURS FOR A STRAIN IN THE 12 DIRECTION AT AN ANGLE THETA OF  45.00
DEGREES IN LAYER  1.

 IT IS LOCATED AT X = 22, Y =  4, AND HAS A VALUE OF    24.87

1THE MINIMUM RESERVE FACTOR OCCURS FOR A STRAIN IN THE   2- DIRECTION AT AN ANGLE THETA OF   0.00 DEGREES IN LAYER  3.

 IT IS LOCATED AT X =  4, Y = 22, AND HAS A VALUE OF  15.53
```

EXAMPLE 4. Sandwich panel with orthotropic facings with the following configuration 45/0/45/core/45/0 from outside to inside. The panel is simply supported in 3 edges and free in the remaining edge representing typical condition for a trailing edge panel. Maximum deflection under cruise condition ($q = 0.00081$ MPa) is calculated to identify minimum stiffness required and a stress analysis performed for strength (maximum strain criterion). The faces are made of carbon fiber fabric and the core is honeycomb aramid paper. Maximum deflection is 0.224 mm.

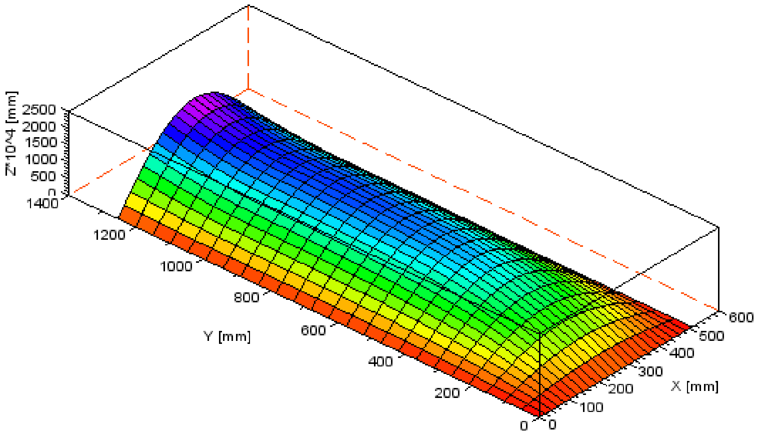


Figure 8. Example 4 deformed shape.

EXAMPLE 5. Curved shell made of monolithic carbon composite, reinforced with stringers[3]. The carbon fiber layers have the following properties: E1 = 146535 MPa, E2 = 9720 MPa, G12 = 6054 MPa, ν12 = 0.34 and a ply thickness of 0.125 mm. The panel modeled in this example has the following dimensions: a x b = 660 x 419 mm and an internal radius of 947 mm. Lay-up is (90/45/-45/0/0/-45/45/90), hence total thickness of skin is 1.00 mm. Regarding the stringers, there are 4 equal stringers with the following dimensions (T-cross section). The stringer lay-up is (+45/-45/+45/-45/+45/-45/0/0/0/0/0/0)S.

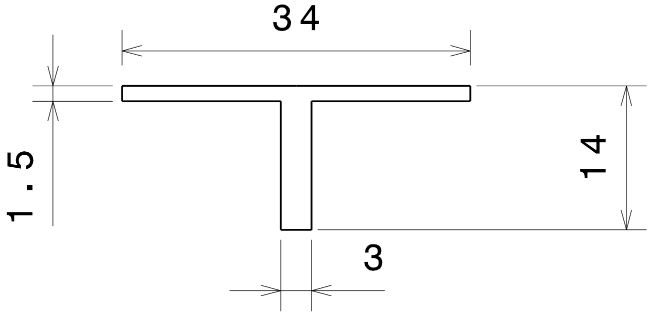


Figure 9. T-stringer cross section dimensions.

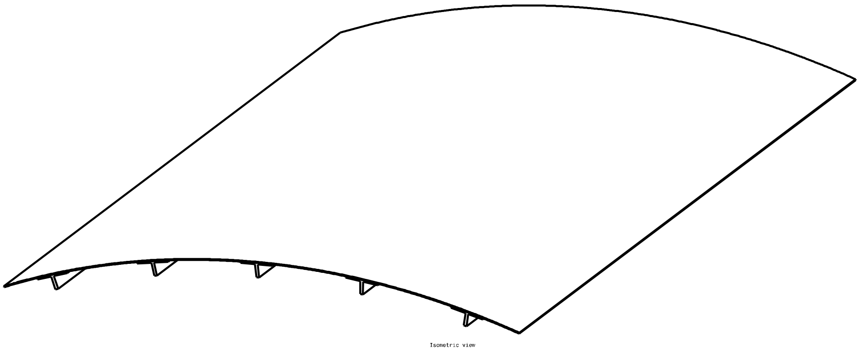


Figure 10. P14 composite shell geometry[3].

Stringer mechanical properties (UNITS = N, mm)

E = 84476.63 MPa

G = 21877.22 MPa

Cross section data (moments are calculated with respect to line of attachment. See Figure 9)

$\bar{y}$= +0.000 mm

$\bar{z}$= -4.216 mm

A = 88.50 mm^2

I_{yy} = 3129.8750 mm^4

I_{zz} = 4941.1250 mm^4

I_{yz} = 0 mm^4

---

[3] R. Zimmermann, H. Klein, A. Kling, Buckling and postbuckling of stringer stiffened fibre composite curved panels – Tests and computations, *Composite Structures*, Volume 73, Issue 2, May 2006, Pages 150-161, ISSN 0263-8223, 10.1016/j.compstruct.2005.11.050. (http://www.sciencedirect.com/science/article/pii/S0263822305003806)

J = 150.75 mm^4

Hence GJ$_s$ = 3297990.9

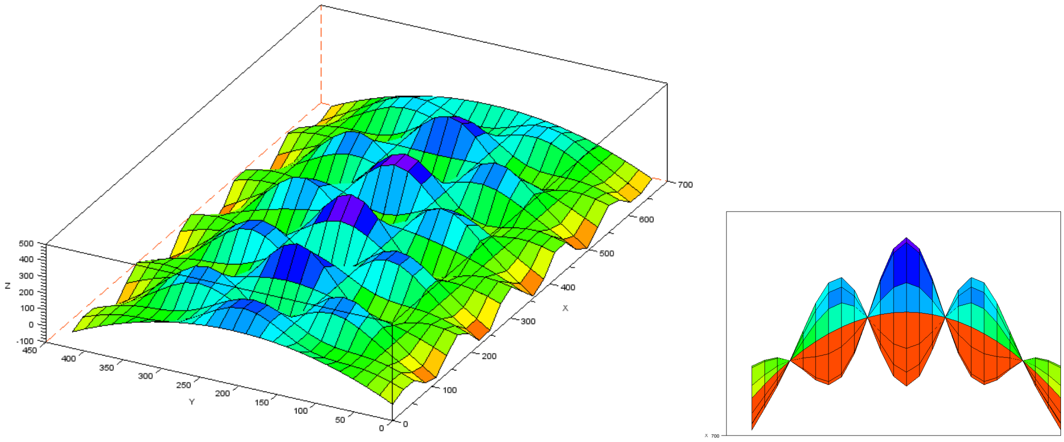


Figure 11. Example 5 initial buckling mode shape (stringer remain in initial position).

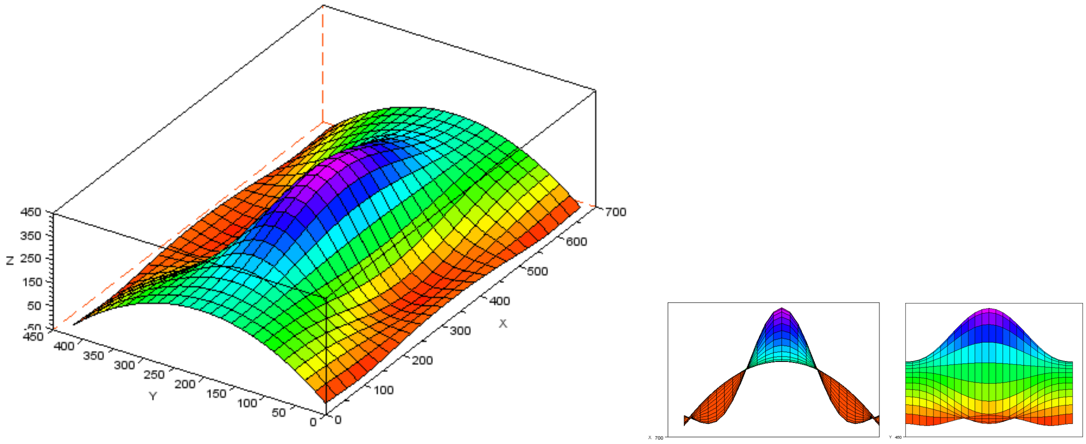


Figure 12. Example 5 general buckling mode shape (stringer bending).

In the following picture a comparison of the front view for the 2 modes can be seen, showing stringer location (b/8 from edge, b/4 distance between stringers)

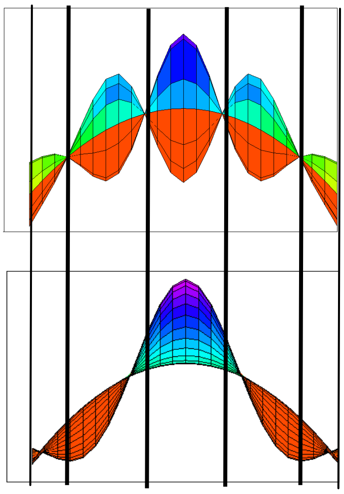


Figure 13. Example 5 general local and general buckling mode shapes compared.

EXAMPLE 6. Monolithic flat plate in carbon fiber composite. Simply supported on the 4 edges is loaded in plane and strains a reserve factors (=allowable strain/applied strain) are calculated for each ply and at extreme fibers.